Pathfinding in 3D Space: A*, Theta*, Lazy Theta* in Octree Structure

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Introduction

Pathfinding addresses the problem of finding shortest paths from source to destination avoiding obstacles. It is an essential component of Machine Intelligence and finds many applications in the fields of robotics, logistics and video games. In particular, pathfinding in 3D space may be useful for drone navigation and real-time 3D strategy games. There exist different algorithms to solve exact shortest paths on graphs (Dijkstra) or 3D surfaces (exact geodesics). However, finding exact Euclidean shortest paths in three or higher dimensions is an **NP-hard problem**. Standard methods for pathfinding in a 2D plane could be extended to 3D, but they are either inefficient in time or memory.

The **objective** of this study is to find approximate shortest paths efficiently in 3D space with obstacles with reasonable memory consumption.

Screenshot from Homeworld

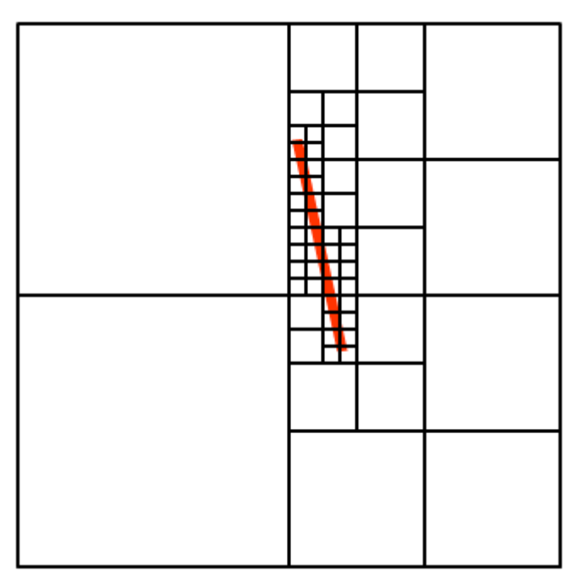
Methods

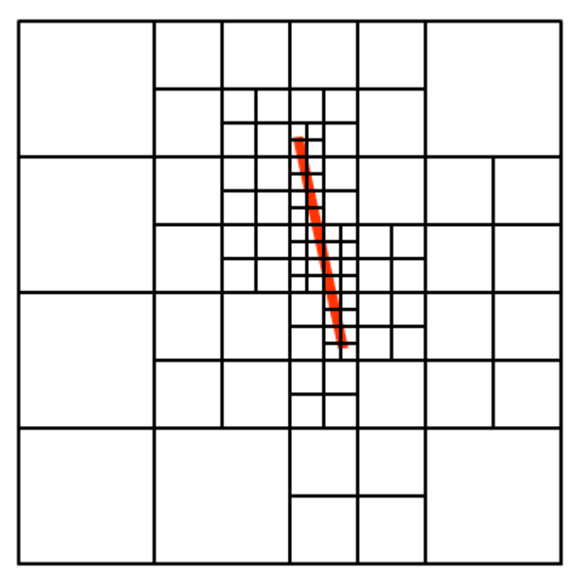
The general **pipeline** of our method is as follows. First, we construct an octree representing the 3D space with obstacles. Second, we construct a graph from the octree. Third, we inject source and destination in the graph. Finally, we apply a graph-based search algorithm to find approximate shortest paths.

Octree construction:

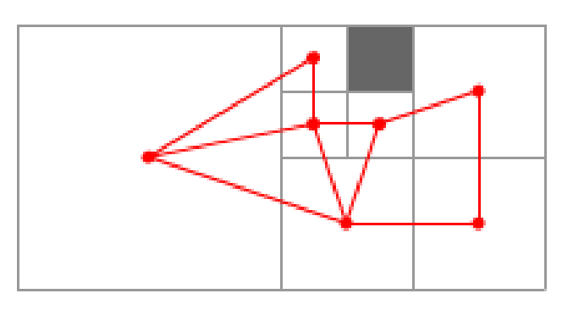
We subdivide the octree recursively wherever the space it represents intersects an obstacle, until a fixed lowest level. Since the obstacles are normally objects represented by triangle meshes, we implement a fast method to detect triangle-cube intersection [1].

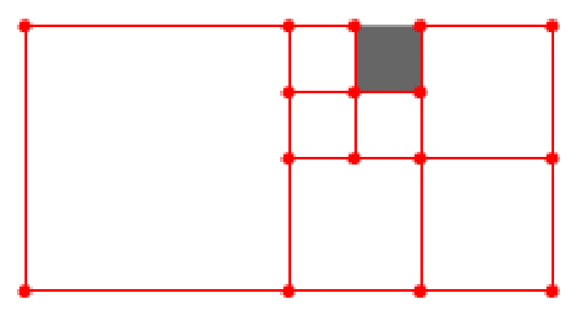
A *progressive octree* is an octree with an additional constraint: we require that the difference of levels between neighbouring octree leaves be no more than 1. This constraint can potentially reduce approximation error while applying graph-based pathfinding algorithms.


Quadtree (Octree in 2D)

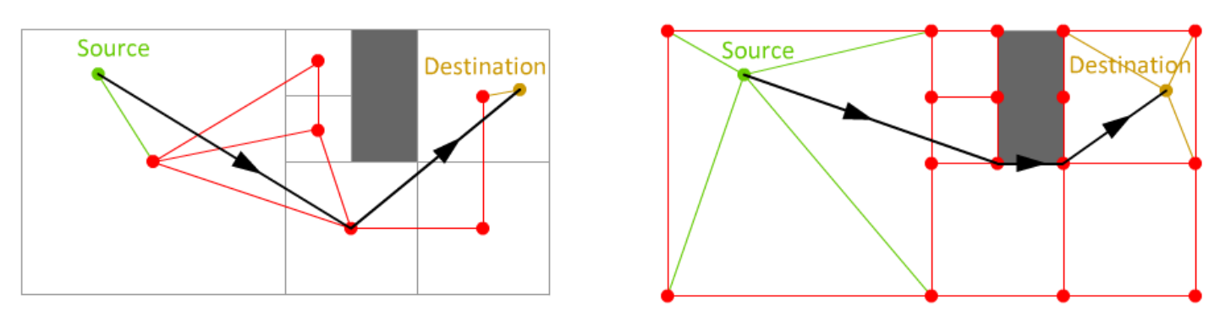

Progressive quadtree

Graph construction:


Dual graph

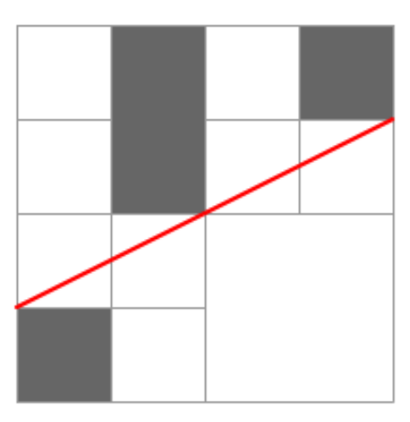

Edge-corner graph

Injection of source and destination:



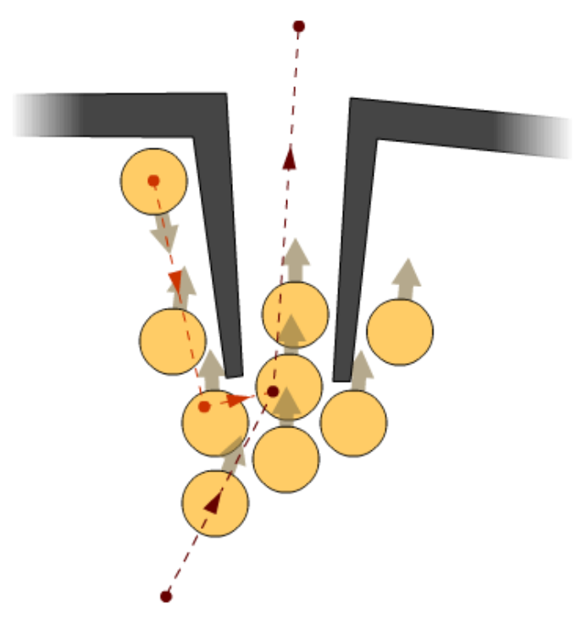
Avoid exhaustive search:
We precompute connectivity using union-find to provide an immediate check of whether a solution exists.

Line of sight:
Integer-based, so not prone to floating point errors



Multisource:
In the case where we have multiple agents all going to the same destination, instead of planning a path for each individually, we start the planning for one agent from the destination and reuse information for the others.

Multi-agent navigation:
Each agent navigates through waypoints. A repulsive force is used to avoid collision. In case an agent loses line of sight to the next waypoint (possibly due to traffic jam), a new path to the next waypoint is replanned.

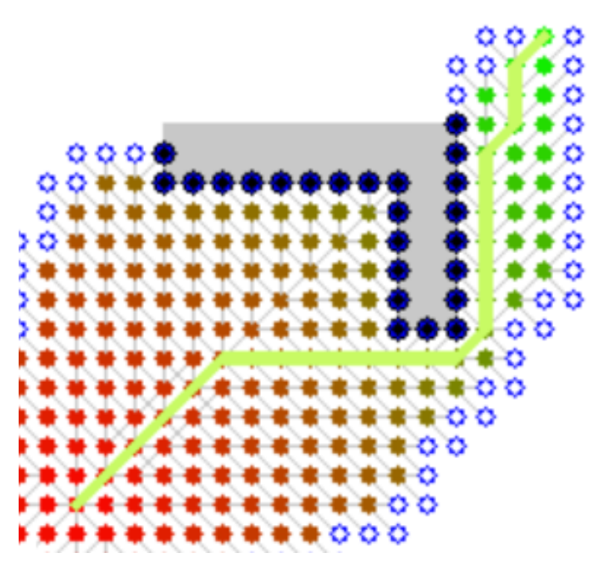


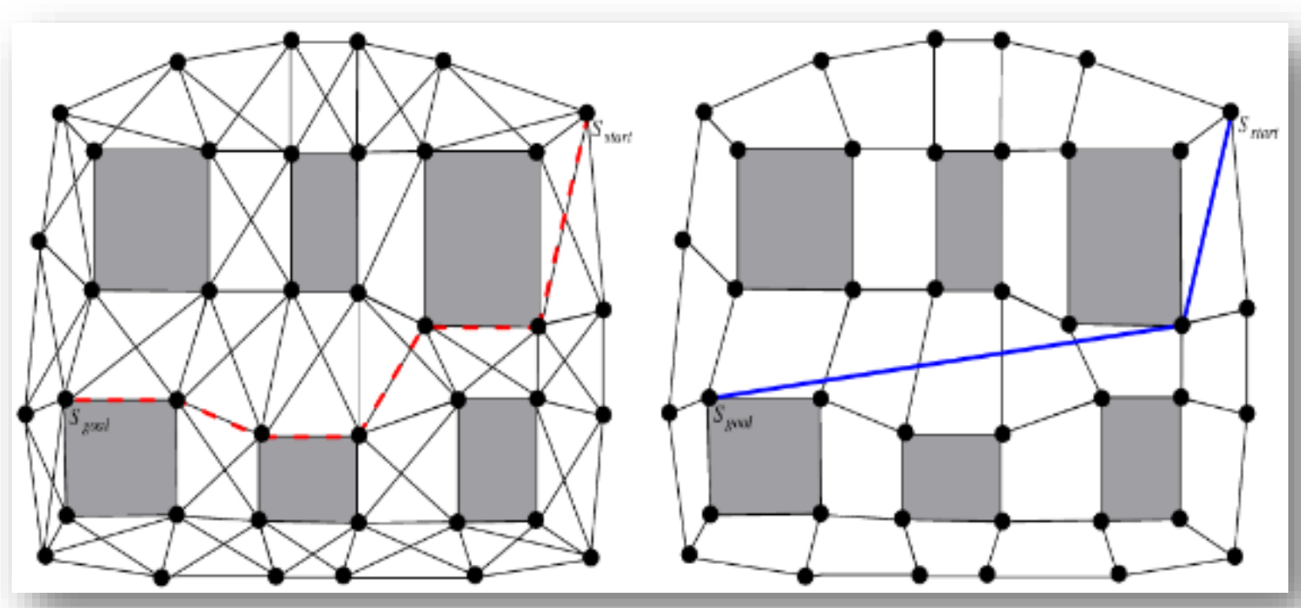
Algorithms

Graph-based search algorithms:

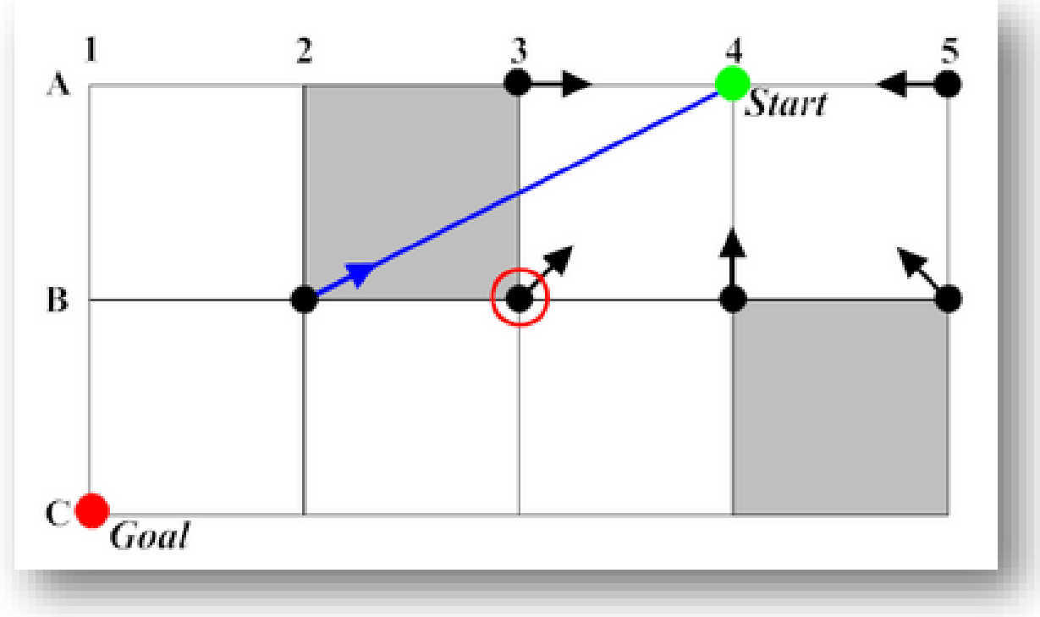
A* (1968 Hart [2]) – generalisation of Dijkstra

It is a best-first search algorithm, meaning that among all possible paths to the destination, it finds an exact shortest path on the graph by first considering the nodes that *appear* to incur the smallest cost (known distance to the source + estimated distance to the destination).


A path found by A*



Theta* (2007 Nash [3]) allows paths outside of the edges by performing a line-of-sight check. However, it does not guarantee to find the shortest path in Euclidean space.

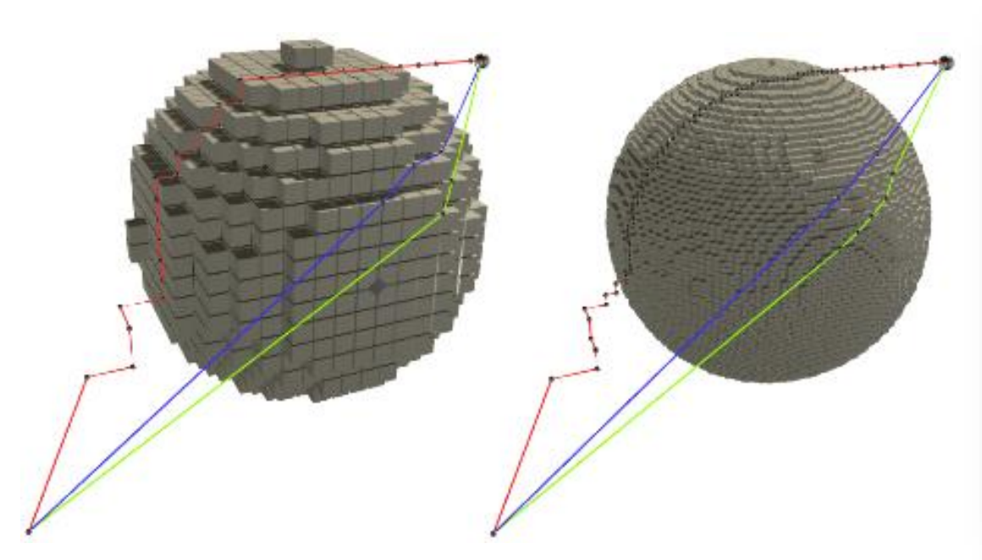


Lazy Theta* (2010 Nash [4]) checks whether the parent of a node is in its line of sight only before exploring its neighbours. It speeds up Theta*, but might find a slightly longer path.

Results

Comparison of data structures and algorithms:

Red: A*,
Green: Theta*,
Blue: Lazy Theta*

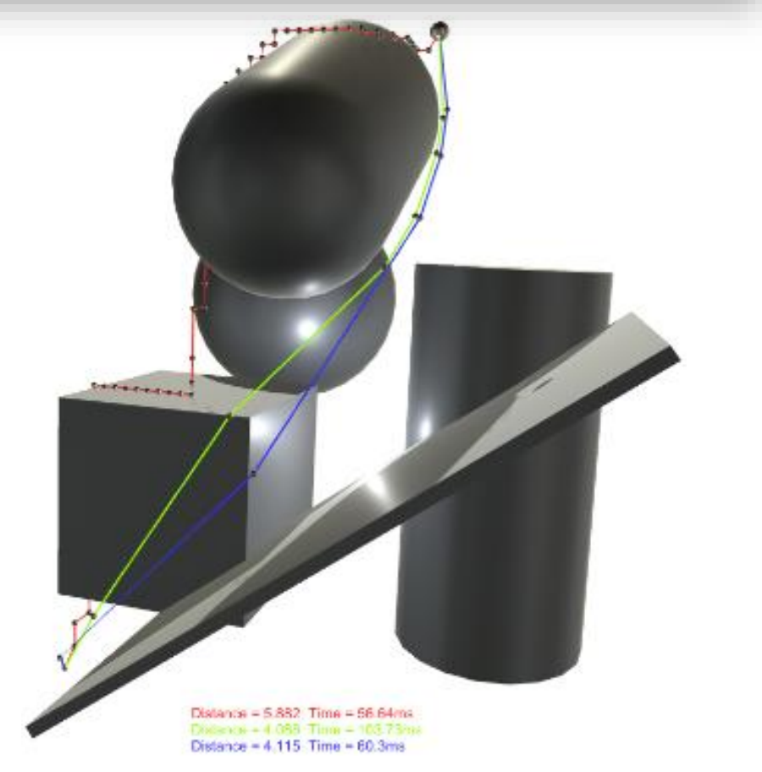


Data Structure	Algorithm	distance	time cost	distance	time cost
Octree	A*	121.38%	1.5ms	121.03%	28.0ms
	Theta*	104.59%	6.7ms	102.71%	236.1ms
	Lazy Theta*	104.65%	4.2ms	102.34%	114.8ms
Progressive Octree	A*	127.43%	3.3ms	126.88%	55.6ms
	Theta*	103.49%	6.3ms	101.23%	229.4ms
	Lazy Theta*	103.68%	3.2ms	101.16%	108.8ms

Table 1: Comparison - A single sphere - Left: level = 7/ Right: level = 9

Data Structure	Algorithm	distance	time cost	distance	time cost
Octree	A*	3.2472	9.5ms	3.3302	5.3ms
	Theta*	2.4108	23.9ms	2.4600	45.5ms
	Lazy Theta*	2.4135	9.5ms	2.4592	16.3ms
Progressive Octree	A*	3.3949	14.1ms	3.3222	7.15ms
	Theta*	2.4009	17.59ms	2.4158	43.1ms
	Lazy Theta*	2.4057	7.78ms	2.4205	14.6ms

Table 2: Comparison - A complex scene - Left: edge-corner graph/ Right: dual graph

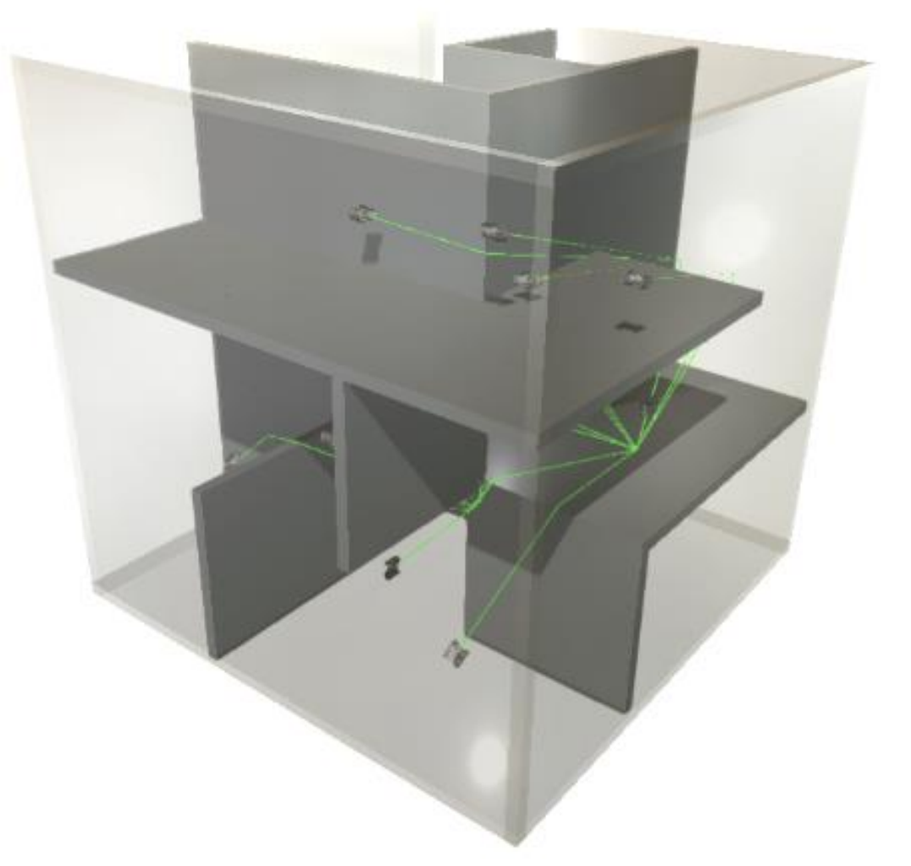


We compare the path distance and the time cost, for combinations of different space discretisation data structures, graph construction methods and graph-based search algorithms, in different obstacle settings.

Conclusion

We proposed methods to find approximate short paths in 3D space efficiently. We came up with a new space discretisation data structure – progressive octree. Our best combination is progressive octree combined with edge-corner graph, using Lazy Theta* for search. We achieved short paths in acceptable error (<2%) for reasonable resolution in all of our test cases with known exact solution. Further optimisation is done to reduce time cost, including considering connected components and reusing information for multisource case.

Possible improvements consist of distributing computation at each frame, using other possible heuristics in A*-family algorithms, post-process with local optimisation, etc.



For more information:

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https://ascane.github.io/

Unity demo

References
[1] Tomas Akenine-Möller. Fast 3d triangle-box overlap testing. In *ACM SIGGRAPH 2005 Courses*, page 8. ACM, 2005.
[2] Nilsson Hart and Raphael. A formal basis for the heuristic determination of minimum cost paths. *IEEE Transactions on Systems, Science, and Cybernetics*, SSC-4(2):100–107, 1968.
[3] Alex Nash, Kenny Daniel, Sven Koenig, and Ariel Felner. Theta*: Any-angle path planning on grids. *Proceedings of the National Conference on Artificial Intelligence*, 22(2):1177, 2007.
[4] Alex Nash, Sven Koenig, and Craig Tovey. Lazy theta*: Any-angle path planning and path length analysis in 3d. In *Proceedings of the National Conference on Artificial Intelligence*, 2010.